Wen-Tsun Wu's Academic Career

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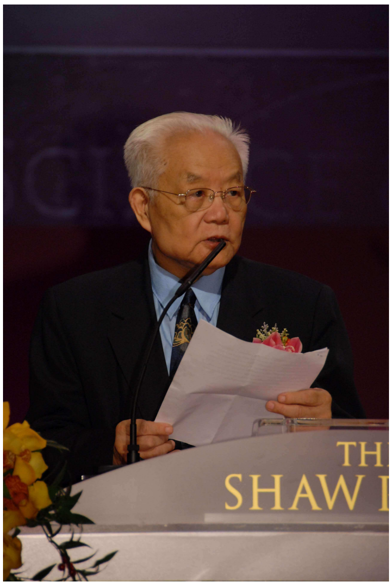

Wen-Tsun Wu

Wu Wen-tsun was born in Shanghai, China on May 12, 1919. He got his B.S. degree in mathematics from Communications University at Shanghai in 1940. Due to warfare, he taught for years in junior middle schools since his graduation from university which caused a complete stop of further learning of mathematics. After the end of Sino-Japanese war in 1945, he became an assistant of the Temporary University, the provisionary re-established Chiao-Tung University, and he was able to revive his mathematical studies. In the summer of 1946, Wu met S.S. Chern who was at that time in charge of the newly established Institute of Mathematics belonging to the Chinese Academy of Sciences. It turns out that this meeting was decisive for the future career of Wu in mathematics.

Chern admitted Wu as one of the young research assistants in his institute, all learning algebraic topology under his guidance. Wu began to access to modern mathematics, and learned algebraic topology including then yet nascent theory of fiber bundles and characteristic classes. One year after Wu brought out his first paper about a simple proof of the product formula of sphere bundles discovered by Whitney for which the original proof was very complicated and intricate and had never published. Under recommendation Chern, this new proof was published in Annals of Mathematics.

In 1946, Wu passed the national examination for sending students abroad and in 1947 Wu was sent to study mathematics in France according to a Sino-France exchange program. Wu went to Strasbourg to study under Ehresmann. It was highly fortunate for Wu in that Ehresmann was one of the founders of fiber bundle theory and also a specialist of Grassmannian varieties so that Wu can continue his previous studies under Chern.

In 1949 Wu completed his National Doctor Thesis which was a somewhat detailed study of characteristic classes via Grassmannian varieties and was published later in 1952. It seems that the terminologies of Pontrjargin classes and Chern classes appeared the first time in his thesis.

In 1949, Wu went to Paris to study under Prof. H. Cartan in CNRS. During his stay in Strasbourg Wu made acquaintance with R. Thom who was at that time a student of Cartan but, being at Strasbourg, took much contact with Ehresmann too. There were a lot of discussions between Thom and Wu. The collaboration was a very fruitful one. In early months of 1950, Thom discovered the topological invariance of Stiefel-Whitney classes and began to found a theory of cobordism while Wu, with aids from H.Cartan, discovered the classes and formulas now bearing his name.

In 1951, Wu returned back to China, became first a professor in Peking University, then one in Academia Sinica from 1953 onwards, and continued his works on Pontrjargin classes. In year 1953, Wu discovered a method of constructing topological invariants of polyhedra which are non-homotopic in character. With these new tools, Wu made a somewhat systematic investigation of classical topological but non-homotopic problems which became ignored at that time owing to the rapid development of homotopy theory. Ultimately, Wu found successful applications to imbedding problems which are typically of topological but non-homotopic character. Wu introduced the notion of imbedding classes, and established a theory of imbedding, immersion, and isotopy of polyhedra in Euclidean spaces which was published in book form later in 1965. Owing to these works on characteristic classes and imbedding classes, Wu was awarded one of the three national first prizes for natural sciences in 1956 and became an Academician of the Chinese Academy of Sciences. He was invited to give an invited lecture in the 1958 International Congress of Mathematicians, although he was unable to attend the congress due to various reasons.

In 1967 onwards, Wu extended and applied his imbedding theory to the practical layout problem of integrated circuits, giving a criterion for the planarity of linear graphs in the form of solvability of some system of linear equations on mod 2 coefficients. Moreover, it gave also methods of actually imbedding an imbeddable graphs in the plane, which seems to be never touched upon by any one else.

In 1965, Wu discovered a simple computational method of defining generalized Chern classes and Chern numbers of an algebraic variety with arbitrary singularities via composite Grassmannians. The papers of Wu were published in Chinese and were little known outside China. The study was unfortunately interrupted by cultural revolution. Only late in 1986, Wu was able to take up this subject again and proved by simple computations the extension of the so-called Miyaoka-Yau inequalities between Chern numbers of algebraic manifolds of dimension 2 to algebraic surfaces with arbitrary singularities. A large number of inequalities as well as equalities among the generalized Chern numbers of algebraic varieties with arbitrary singularities have henceforth be discovered by means of this computational method of Wu by his colleagues.

During the cultural revolution, Wu was sent to some factory manufacturing analogue computers and also some kind of hybrid computer. Wu was striken the first time by the power of a computer. Wu devoted also one year or so to the study of Chinese ancient mathematics. Wu began to understand what Chinese ancient mathematics really was and was greatly striken by the deepness and powerfulness of its thoughts and its methods. Wu went on to point out that a majority of Chinese ancient mathematics was to develop algorithmic methods for solving various problems and in particular solving algebraic equations, and the constructive approach of the Chinese ancient mathematics in contrast to the Euclid's axiomatic approach also had deep influence on the developments of mathematics. Wu was invited for the second time to give an invited lecture in the 1986 International Congress of Mathematicians to report his discoveries about the Chinese ancient mathematics.

It was under these two influences that Wu tried in the end of 1976 to seek the possibility of proving geometry theorems in a mechanical way. After several months of trials, Wu ultimately succeeded in developing a method of mechanical geometry theorem proving. This method has been applied to prove or even discover hundreds of non-trivial difficult theorems in elementary and differential geometries on a computer in an almost trivial manner. Due to this work, Wu was awarded the Herbrand Award for Distinguished Contributions to Automated Reasoning in 1997, which is considered the highest award in the field of automated reasoning.

The last discovery marks the second turning point in Wu's scientific life, the first one being his meeting with Chern. Since that time, Wu has completely changed his directions of research and concentrated his efforts in extending the method in various directions, both theoretical and practical, aiming at what he has called mechanization of mathematics. In particular, based on the classic work of Zhu in the fourteenth century and the Ritt's differential algebra, he established a method for solving algebraic and differential equations by transforming an equation system in the general form to equation systems in triangular form. In view of its future bright prospects, he was offered in 1990 special funds to establish the Mathematics Mechanization Research Center, later became the Key Laboratory of Mathematics Mechanization.

Wu's research covers topology, computer mathematics, algebraic geometry, game theory, and history of Chinese Mathematics. A detailed introduction to Wu's research work in topology and computer mathematics is described below.

(1) Algebraic topology

Topology mainly deals with continuity of geometric objects and is one of the fundamental bases for many important mathematical branches. Wu's work lies in his introduction of Wu's imbedding and characteristic classes and the establishment of Wu's formulae. Characteristic classes are basic invariants depicting fiber bundles and manifolds. This concept has been developed since the end of 1940s by many celebrated scholars, but their work was mostly descriptive. Wu simplified their work and systematized the theory. For example, there was Stiefel-Whitney characteristic class, Pontryagin's characteristic class and Chern's characteristic class. Wu completed a deep-going analysis of their relations and proved that other characteristic classes might be derived from Chern's classes but vice-versa was untrue. In addition, he introduced new methods and means. In the field of differential manifold, he introduced Wu's characteristic class, not only being an abstract concept but also computable in concrete questions. He establsihed formulae to express the Stiefel-Whitney class by Wu's classes and the relations among them. His work gives rise to a series of important applications, (jointly with others) making the theory on characteristic classes an enriched and perfect chapter in topology. Imbedding theory in topology is to study the realization of complicated geometric objects in Euclidean space. Before Wu's work, there had been some incomplete results in this aspect. He put forward his series of topological invariants of Wu's imbedding classes, probing into the core problems of the subject and developed a unified theory of imbedding. In later years, he applied this work to the layout problem in electric circuits, leading to a new criterion for imbedding of a planner diagram. In nature, his new theory is different from all of past theories because it is computable. Wu's work in topology is original, inspiring and precursory, triggering a great deal of succeeding work of other mathematicians and leading to a host of monumental achievements in the discipline. For example, among the laureates of the Fields prize, the highest award for mathematicians around the world, five (R.Thom, J.Milnor, S. Smale, M.F. Atiyah, E. Witten) cited his work, including three using Wu's results in their prize-winning papers. Up to now, some of Wu's work has become classic results in topology and written in college textbooks. Another example for this statement is an article of Henri Cartan, winner of the Wolf Prize, in which he says: the naming of the so-called Cartan's Formula is improper because, although its demonstration was given by him, the formula itself actually had been first suggested by Wu. Another winner of the Wolf Prize, German mathematician F. Hirzebruch transplanted Wu's formula about 4D-manifold Pontryagin's characteristic classes into his own work, leading to the proof of the high-dimension Riemann-Roch Theorem. Before the debut of Wu's work, there exist unsurmontable obstacles in the computation of homology and homotopy groups. Just because of his work, the Steenord computation and characteristic classes got married in an ingenious way and since then, the integration of algebraic topology with other mathematical branches becomes possible, which leads to the emergence of many new research realms, such as Hopf's algebraic theory, manifold topology and generalized homology.

(2) Computer mathematics

Wu is known in the automated deduction community for the method he formulated in 1977, marking a breakthrough in automated geometry theorem proving. Geometry theorem proving was first studied in the 1950s by Herbert Gelernter and his associates. Although some interesting results were achieved, the field saw little progress for almost twenty years, until "Wu's method" was introduced. In few areas of automated deduction can one identify a specific person who turned the field around completely. Wu is clearly such a person. His work started from the observation of the correspondence between plane geometry and analytic geometry. Specifically, one can transform a problem in elementary geometry into a set of polynomials and, by solving the polynomials, deduce the correctness of the theorem. This transformation of problems in geometry into problems in algebra enables researchers to use a full range of algebraic tools, which are much easier to automate than their counterparts in geometry. His method can be used not only to prove theorems in geometry, but also to discover theorems and to find degenerated cases automatically. Wu started his research on geometry theorem proving in 1976. S.C. Chou introduced Wu's method to the international community of automated deduction while studying at the University of Texas in the early 1980s. Chou's prover proved several hundred theorems and further demonstrated the power of Wu's method. Geometry theorem proving was by then fully revised and became one of the most actively researched and successful areas in automated deduction. Wu continued refining and extending his method and added a dazzling array of application domains whose proofs can be automated. They include plane geometry, algebraic differential geometry, non-Euclidean geometry, affine geometry and nonlinear geometry. Not limiting the applications to geometry alone, he also gave mechanical proofs of Newton's gravitational laws from Kepler's laws, and many new results from the fields of CAGD, machinery design, chemical equilibrium, celestial mechanics. His work turned theorem proving from one of the less successful research areas in automated deduction to one of the most successful. Indeed, there are few areas for which one can claim that machine proofs are superior to human proofs. Geometry theorem proving is such an area.

Wu went on to broaden the range of his work on theorem proving by calling for the study of mathematics mechanization, which is the effort to mechanize the mental labor in the field of mathematics, in particular, theorem proving, discovering, and equation solving. In 1979, Wu proposed the "Program of Mathematics Mechanization" which consists of the following aspects: (1) Cover as much as possible the whole of mathematics by domains each of which is sufficiently small to be mechanizable, at the same time also sufficiently large to contain lot of theorems or problems of high mathematical interest. (2) Apply the methods of mathematics mechanization to interdisciplinary studies and engineering problem solving. The central theme of mathematics mechanization in the past thirty years is to study effective symbolic methods for solving various equation systems. Developing methods for solving equations is the main concern of mathematics in ancient China. A peak of the work of ancient Chinese mathematicians on equation solving is Szejie Zhu's work in year 1303, which gives a quite general method for solving equation systems with four indeterminates. It is along the lines of thought furnished by classic work of the ancient Chinese mathematics and the modern work of Ritt on Differential Algebra, Wu established the Ritt-Wu zero decomposition theorem for algebraic and differential equations. Wu's work on mechanical theorem proving and equation solving in the late 1970s and early 1980s inspired a great deal of studies on these issues and had been applied to many fields including computer vision, image processing, intelligent CAD, CAGD, hardware verification, coding theory and cryptography, robotics and mechanisms, etc.

Key Laboratory of Mathematics Mechanization,
Chinese Academy of Sciences
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Wu Wen-tsun received The Shaw Prize in Mathematical Sciences 2006 jointly with David Mumford.

The official press release can be found at The Shaw Prize Foundation webpage
`http://www.shawprize.org/`

The previous recipients of the The Shaw Prize in Mathematical Sciences include Andrew John Wiles (2005) and Shiing-Shen Chern (2004).