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Wen-Tsun Wu: His Life and Legacy

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Professor Wen-Tsun Wu passed away on May 7, 2017, a few days ahead of his 98th birthday. Wu is one of the most influential Chinese mathematicians and computer scientists. In his early years, he made fundamental contributions to algebraic topology. In the late 1970s, Wu proposed the Wu's method of geometric theorem proving and founded the school of Mathematics Mechanization. His study on the ancient Chinese mathematics opened a new era in the field of history of Chinese mathematics. He also made contributions in the area of algebraic geometry, invariant theory, complex geometry, and game theory. His legacy consists not only of his academic work but also of several generations of scientists who learned from him the joy and enthusiasm of scientific research and the way to go about it.

Early Years

Wu was born in Shanghai, China on May 12, 1919. His father was an editor and a translator in a publishing company. As the only surviving son of the family, Wu received much attention from his father and mother both in life and in education.

During the 1920s and 1930s, Shanghai rapidly changed into the most modern city in China, and arguably in Asia. Social unrest led to several interruptions and change of schools in his primary and junior high school years. In 1933, Wu went to a private high school and received very good education. For instance, the second-year algebra course was taught by a teacher who was educated in Japan and the textbook was "A College Algebra" by Henry Burchard Fine. It should be noticed that the private school was founded by a businessman and the tuition was actually very low.

In 1936, Wu graduated from high school, and the school offered him a scholarship for university education. Wu himself preferred to learn physics, but the scholarship was for mathematics major only. Unable to pay the university tuition from his family, Wu accepted the offer and went to the mathematics department of Shanghai Jiao Tong University, which is one of the best Chinese universities. He received his B.S. degree in mathematics from Shanghai Jiao Tong University in 1940.

From 1937 to 1945, China was at war with Japan. Shanghai was occupied by Japan and the Chinese government and most universities (or parts of them) moved to the west part of China. As the only son of the family, Wu was asked by the family to stay in Shanghai. Unable to find a job in universities, Wu taught mathematics in high schools from 1940 to 1946. He tried to do research in general topology and elementary geometry. He once showed his results to algebraic geometer W.L. Chow who was at that time doing business in Shanghai. Chow told Wu frankly that Wu has strong ability, but the research topic was meaningless.

The First Research Result

In 1946, Wu was accepted by Shiing-Shen Chern as one of his research assistants in the newly established Institute of Mathematics belonging to Academia Sinica. This is the turning point in Wu's life. Chern, a great differential geometer, taught algebraic topology to a group of young scholars in the institute. Wu began studying modern mathematics, and learned algebraic topology including the nascent theory of fiber bundles and characteristic classes at that time.

Only one year later, Wu published his first paper in the Annals of Mathematics, about a simple proof of the product formula of sphere bundles discovered by the great topologist Whitney [1]. The product formula is a key result in the theory of characteristic classes. It is said that Whitney's original proof was quite complicated and he planed to publish a book on this topic. After reading the proof of Wu, Whitney said that he need not to keep his proof any more.

Years in France and Characteristic Class

In 1946, the Chinese government established a scholarship for a Sino-France exchange program. Wu took the national examination and received the precious scholarship. In 1947, Wu went to Strasbourg to study under Charles Ehresmann, who was one of the founders of fiber bundle theory and also a specialist of Grassmannian varieties. Wu completed his National Doctor Thesis in 1949 which was a detailed study of characteristic classes via Grassmannian varieties. His thesis was later published as a book by Hermann & Cie in 1952, jointly with his classmate Reeb [5].

Wu's thesis is one of the first comprehensive studies of characteristic classes and contains essentially new results. The terminology of Pontrjargin classes and Chern classes appeared for the first time in Wu's thesis. To clarify some of the results, topologist Hopf went to Strasbourg to clarify whether some of Wu's results are correct. As a by product, Wu proved that spheres of dimension $4k$ have no complex structure, which was the first nontrivial result on the complex structure of spheres [2]. Until now, it is still an open problem whether the sphere of dimension 6 has complex structure.

In 1949, Wu moved to Paris to study under H. Cartan at the CNRS. This is one of the high creation period of Wu. Wu discovered the classes and formulas about the characteristic classes of fiber bundles and manifolds, now bearing Wu's name [3, 4].

Characteristic classes are basic invariants depicting fiber bundles and manifolds. This concept had been developed since the end of 1940s by many celebrated scholars, but their work was mostly descriptive. Wu simplified their work and systematized the theory. Wu completed a deep-going analysis of the relations among Stiefel-Whitney characteristic class, Pontryagin's characteristic class, and Chern's characteristic class, and proved that other characteristic classes can be derived from Chern's classes. He also introduced new techniques, for example, in the field of differential manifold, he introduced Wu's characteristic class, not only being an abstract concept but also computable. He established formulae to express the Stiefel-Whitney class by Wu's classes. Wu's work led to a series of important applications, thus enriching the theory on characteristic classes.

Algebraic topology was founded by French mathematician H. Poincaré in early 1900s. By the time Wu went to France, the center of topology is USA and France is left behind in this this field. During the early 1950s, several young mathematicians working French made fundamental contributions to topology and their work pushed France to the forefront of the field of topology again. These young mathematicians include A. Borel, J.P. Serre, R. Thom, and Wu, among whom Serre, Thom, and Wu were students of H. Cartan.

During his stay in Paris from 1949 to 1951, he attended the famous Bourbaki seminar held in Paris since 1948, which had a profound influence on him.

Embedding Class and the First National Prize

In 1951, Wu returned to China, was appointed first a professor in Peking University, and later in the Chinese Academy of Sciences from 1953 onwards.

Wu continued his work on Pontrjargin classes. In 1953, Wu discovered a method of constructing topological invariants of polyhedra which are non-homotopic in character. With these new tools, Wu made a systematic investigation of classical topological but non-homotopic problems. They did not get much prominence partly owing to the rapid development of homotopy theory during that time. Later, Wu found successful applications to embedding problems which are typically of topological but non-homotopic character. Wu introduced the notion of embedding classes, and established a theory of embedding, immersion, and isotopy of polyhedra in Euclidean spaces which was summarized as a book [7].

Wu was awarded one of the three national first class prizes for natural sciences in 1956 and became a member of the Chinese Academy of Sciences in 1957 because of his fundamental contributions to characteristic classes and embedding classes. He was invited to give an invited lecture at the 1958 International Congress of Mathematicians (ICM), which he was unable to attend.

Marriage and Family

In 1953, Wu married Ms. Peihe Chen and they raised three daughters Yueming Wu, Xingxi Wu, Yunqi Wu, and one son, Tianjiao Wu. Ms. Peihe Chen has provided tremendous support to Wu. She not only took all the household work but also typewrite Wu's English monographs.

USTC and Algebraic Geometry

In 1958, The Chinese Academy of Sciences established the University of Science and Technology of China (USTC) and the prominent scientists from the Academy were responsible for the education of the USTC. Wu was in charge of class 1960 of mathematics major in USTC. He taught calculus for one and half a year using the text book of I.M. Ostroski. Starting from the third year, the students were divided into specialized groups and Wu was in charge of the specialized group in topology and algebraic geometry.

At that time, algebraic geometry is basically empty in China and it is Wu who introduced this field into China. In 1965, Wu discovered a simple computational method of defining generalized Chern classes and Chern numbers of an algebraic variety with arbitrary singularities via composite Grassmannians [8], which is an important open problem at that time. Wu's papers on these topics were published in Chinese and were little known outside China. His research was unfortunately interrupted by the cultural revolution started in 1966. In 1986, Wu was able to take up this subject again and proved by simple computations, the extension of the so-called Miyaoka-Yau inequalities between Chern numbers of algebraic manifolds of dimension 2 to algebraic surfaces with arbitrary singularities. A large number of inequalities as well as equalities among the generalized Chern numbers of algebraic varieties with arbitrary singularities have been discovered since by means of this computational method.

Game Theory and Planarity of Linear Graph

Since late 1950s, Wu's research on algebraic topology was unfortunately stopped due to political reasons, in particular, the cultural revolution from 1966 to 1976. Starting in 1958, the Chinese government emphasized linking theory with practice, and applied mathematics such as computational mathematics, partial differential equations, statistics were listed as the priority fields. In the operational level, the policy went to extreme: researches on pure theory were completely stopped for quite a long time. Wu was assigned to the Operations Research group of the Institute of Mathematics. For a while, Wu felt a sense of hesitation and uncertainty, and this must be painstaking time for him. But, he quickly picked up research first in game theory and then in plane embedding of integrated circuits and left his marks in each of these topics.

In 1929, J von Neumann established the cooperative game theory. A breakthrough in game theory is the non-cooperative game established by J. Nash. One problem with the non-cooperative game is that the equilibrium points are not unique. In 1962, Wu and his student J.H. Jiang proposed the concept of essential equilibrium points for non-cooperative game and proved its existence [6], which was the earliest work on the important topic of perfect Nash equilibrium.

In 1967, Wu extended and applied his embedding theory to the practical layout problem of integrated circuits, giving a criterion for the planarity of linear graphs in the form of solvability of some system of linear equations on mod 2 coefficients [9]. This work also resulted in methods of actually embedding an embeddable graphs in the plane, a problem which seems to have not been studied by anyone else at that time.

Return to Topology: I^*-functors

Beginning in 1971, the political environment in China was gradually changed for the better side. In particular, foreign exchanges with the West started in 1972. In 1973, topologist Franklin Paul Peterson visited Beijing and brought Wu's attention of the "rational homotopy theory" created by D. Sullivan. Wu turned the rational homotopy theory into algorithmic form, introducing a new terminology called I^*-functors. These results were later summarized in a book [14]. This is Wu's last piece of work on topology.

It should be noticed that one main feature of Wu's research work is the constructive style, first appeared in his work of characteristic classes and then in the work of I^*-functors. The importance of making an abstract concept computable goes without saying. In his work on mathematics mechanization, algorithm becomes the central issue of his research.

History of Ancient Chinese Mathematics

In 1974, Wu began to study the ancient Chinese mathematics. Wu observed that most of the ancient Chinese mathematics are constructive or algorithmic, and in particular, systematic methods for solving multivariate algebraic equations were given. In a paper published in 1975, Wu argued that the constructive approach of the ancient Chinese mathematics had equally important influence on the development of mathematics as that of Euclid's axiomatic approach.

To support his idea on the role played by the algorithmic methods on the development of mathematics, Wu began to seriously study the history of Chinese mathematics. Before Wu, the main focus of the study of history of Chinese mathematics is to discover what mathematics was done in ancient China, with the aim of answering the question of what mathematical science, if any, existed in ancient China. Wu led studies in the history of mathematics in China on to a new era, that of recovering how mathematics was done in ancient China and how the ancient Chinese mathematics influenced the development of mathematics in the world.

Wu's studies initiated from an ancient classic Nine Chapters of Arithmetic (dated about 100 B.C.) and its Annotations by Liu Hui (in A.D. 263). Wu compared the various scripts and reconstructed the proofs of Liu's theorem in an ancient style, quite different from the usual Euclidean approach. Based on the research of him and his followers, he believed that Chinese mathematics has its own tradition, which is based on computation as opposed to the axiomatic and proof approach of Western mathematics. To some extent, Wu's own work on geometry theorem proving is a demonstration of how East meets West and how the two trends of thought complement each other.

Wu was invited for the second time to give an invited lecture on the development and history of the ancient Chinese mathematics at in the 1986 International Congress of Mathematicians [13].

Automated Geometry Theorem Proving and Herbrand Award

Influenced by his findings on the algorithmic feature of the ancient Chinese mathematics, Wu tried at the end of 1976 to seek the possibility of proving geometry theorems in a mechanical way. After several months of trials, Wu ultimately succeeded in developing a method of mechanical geometry theorem proving [10, 11, 12]. Wu's method has been applied to prove and discover hundreds of non-trivial difficult theorems in elementary and differential geometries on a computer.

Automated geometry theorem proving was started in the 1950s as one of the pioneering work of Artificial Intelligence. The main idea is to write Proof Machines that follow how people prove a geometry theorem. The key ideas developed include search heuristics, figure based search, using lemmas, etc. Unfortunately, proof programs based on these methods are rather weak in the sense that only very simple theorems can be proved. On the other hand, Wu's approach is powerful enough to prove almost all geometry theorems. This raised the hope that Wu's method can be applied to much more areas and revived the area of automated geometry theorem proving.

For his work on automated geometry theorem proving, Wu was awarded the Herbrand Award for Distinguished Contributions to Automated Reasoning in 1997, which is considered the highest award in the field of automated reasoning. Here are some of the citations.

> Wu is known in the automated deduction community for the method he formulated in 1977, marking a breakthrough in automated geometry theorem proving.
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> Geometry theorem proving was first studied in the 1950s by Herbert Gelernter and his associates. Although some interesting results were achieved, the field saw little progress for almost twenty years, until "Wu's method" was introduced. In few areas of automated deduction can one identify a specific person who turned the field around completely. His method can be used not only to prove theorems in geometry, but also to discover theorems and to find degenerated cases automatically.
>
> Wu continued refining and extending his method and added a dazzling array of application domains whose proofs can be automated. They include plane geometry, algebraic differential geometry, non-Euclidean geometry, affine geometry and nonlinear geometry. Not limiting the applications to geometry alone, he also gave mechanical proofs of Newton's gravitational laws from Kepler's laws and of problems in chemical equilibrium and robotics. Wu's work turned geometry theorem proving from one of the less successful research areas in automated deduction to one of the most successful. Indeed, there are few areas for which one can claim that machine proofs are superior to human proofs. Geometry theorem proving is such an area.

Mathematics Mechanization and KLMM

Wu's work on automated geometry theorem proving marked the second turning point in Wu's scientific life. Wu completely changed his directions of research and concentrated his efforts in extending his method in various directions, both theoretical and practical, aiming at what he has called the *Mechanization of Mathematics*.

The first industrial revolution in 18th century is the replacement of human labor by machinaries which caused a devaluation of human arm by somewhat *mechanization* of muscle labor. The coming new industrial revolution may be considered as partial replacement of human brain by some kind of machinaries which will devalue the human painstaking brain thinking by somewhat *mechanization* of mental labor. Wu believed that since mathematics is the basis of all science and engineering and is a typical mental labor, mathematics has the superiority, priority, and even easiness of being *mechanized* than any other kinds of mental labors. After summarising what had been said and done by great minds such as Descartes, Leibniz, Hilbert, Goedel, and Norbert Wiener, Wu proposed his *Mathematics-Mechanization Program*:

> Cover as much as possible the whole of mathematics by domains each of which is small-enough to be mechanizable and is however large-enough to have mathematical significance and interest.

Based on the classic work of Shi-Jie Zhu in the fourteenth century and Ritt's techniques in differential algebra, Wu developed a method for solving systems of algebraic equations by transforming an equation system in the general form to a family of equation systems in triangular form, much like Gaussian elimination method for linear equations. Wu also emphasized on using methods of mechanized mathematics to engineering problems and he worked on problems from robotics, computer vision, chemical engineering, mechanics, etc. Most of Wu's work on mathematics mechanization were summarized in the book [15].

Wu was offered in 1990, special funds by the Chinese Academy of Sciences to establish the Mathematics Mechanization Research Center, which later became the Key Laboratory of Mathematics Mechanization (KLMM).

In 2000, the Chinese government established the State Preeminent Science and Technology Award which is issued by the President of the People's Republic of China to scientists working in China. Wu received the first State Preeminent Science and Technology Award due to his work on topology and mathematics mechanization.

Shaw Prize

In 2006, Wu together with David Mumford received the Shaw Prize in Mathematical Sciences, considered to be the Nobel prizes of the East. In a communique, the Shaw Prize Committee stated

> David Mumford and Wu Wentsun both started their careers in pure mathematics (algebraic geometry and topology respectively) but each then made a substantial move towards applied mathematics in the direction of computer science.
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> Mumford worked on computer aspects of vision and Wu on computer proofs in the field of geometry. In both cases their pioneering contributions on research and to the development of the field were outstanding. Many leading scientists in these areas were trained by them or followed in their footsteps.
>
> Wu Wentsun was one of the geometers strongly influenced by Chern Shiing-Shen (Shaw Laureate in 2004). His early work, in the post-war period, centered on the topology of manifolds which underpins differential geometry and the area where the famous Chern classes provide important information. Wu discovered a parallel set of invariants, now called the Wu classes, which have proved almost equally important. Wu went on to use his classes for a beautiful result on the problem of embedding manifolds in Euclidean Space.
>
> In the 1970s, Wu turned his attention to questions of computation, in particular the search for effective methods of automatic machine proofs in geometry. In 1977 Wu introduced a powerful mechanical method, based on Ritts concept of characteristic sets. This transforms a problem in elementary geometry into an algebraic statement about polynomials which lends itself to effective computation.
>
> This method of Wu completely revolutionized the field, effectively provoking a paradigm shift. Before Wu the dominant approach had been the use of AI search methods, which proved a computational dead end. By introducing sophisticated mathematical ideas, Wu opened a whole new approach which has proved extremely effective on a wide range of problems, not just in elementary geometry. Wu also returned to his early love, topology, and showed how the rational homotopy theory of Dennis Sullivan could be treated algorithmically, thus uniting the two areas of his mathematical life.
>
> In his 1994 Basic Principles in Mechanical Theorem Proving in Geometry (Springer), and his 2000 Mathematics Mechanization (Science Press), Wu described his revolutionary ideas and subsequent developments. Under his leadership, Mathematics Mechanization has expanded in recent years into a rapidly growing discipline, encompassing research in computational algebraic geometry, symbolic computation, computer theorem proving and coding theory.

Although the mathematical careers of Mumford and Wu have been parallel rather than contiguous they have much in common. Beginning with the traditional mathematical field of geometry, contributing to its modern development and then moving into the new areas and opportunities which the advent of the computer has opened up, they demonstrate the breadth of mathematics. Together they represent a new role model for mathematicians of the future and are deserved winners of the Shaw Prize.

Memorial Website

A memorial website for Professor Wen-Tsun Wu was established at `http://www.amss.ac.cn/wwj`.

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