Conference: Nonlinear Analysis and its Applications in Geometry, 2023

Organized by: Shibing Chen, Guohuan Qiu, Yi Zhang

Date: July 16 - July 22, 2023
Zoom : 82223332367 Code: AMSS2023 and Offline: AMSS N202

1 Schedule

All times are local to Beijing.

Date	Venue	Time	Speaker	Chair
Sunday, July 16	AMSS	Registration		
Monday, July 17	AMSS	14:30 - 15:00	Opening ceremony	Ping Zhang
		15:00 - 15:45	Yuxin Ge	Xinan Ma
		16:00 - 16:45	Xavier Cabre	
		17:00 - 19:00	Dinner Break	
	Zoom	19:00 - 19:45	Karoly Boroczky	Shibing Chen
		20:00 - 20:45	Zhijie Chen	
Tuesday, July 18	AMSS	15:00 - 15:45	Yong Wei	Guohuan Qiu
		16:00 - 16:45	Qirui Li	
		17:00 - 19:00	Dinner Break	
	Zoom	19:00 - 19:45	Alessio Figalli	Ping Zhang
		20:00 - 20:45	Nicola Fusco	
Wednesday, July 19	AMSS	15:00 - 15:45	Kelei Wang	Yi Zhang
		16:00 - 19:00	Dinner Break	
	Zoom	19:00 - 19:45	Jose Galvez	Liqun Zhang
		20:00 - 20:45	Yannick Sire	
Thursday, July 20	AMSS	15:00 - 15:45	Genggeng Huang	Huaiyu Jian
		16:00 - 16:45	Xu-Jia Wang	
		17:00 - 19:00	Dinner Break	
	Zoom	19:00 - 19:45	Emanuel Milman	Bangxian Han
		20:00 - 20:45	Xavier Ros-Oton	Quochung Nguyen
Friday, July 21	AMSS	15:00 - 15:45	Jie Zhou	Yuan Zhou
		16:00 - 16:45	Zhizhang Wang	
		17:00 - 19:00	Dinner Break	
	Zooms	19:00 - 19:45	Guofang Wang	Youde Wang
		20:00 - 20:45	Jingang Xiong	
Saturday, July 22	Leaving			

2 Titles and abstracts

- **Karoly J. Boroczky** (Renyi Institute of Mathematics, Budapest, Hungary)

  Title: L_p-Minkowski problem - Old and New results

  Abstract: Lutwak's L_p Minkowski problem as a Monge-Ampere equation on the n-dimensional sphere for real p has been in the center of attention the last couple of decades. The talk surveys the state of art (like the recent resolution of the case $p < -n-1$, or stability versions strengthening Brendle, Choi and Daskalopoulos' celebrated result about the uniqueness of the unit ball as a solution for the suitable equation for $p > -n-1$), and points out some major open problems.

- **Xavier Cabre** (ICREA and Universitat Politecnica de Catalunya)

  Title: Stable solutions to semilinear elliptic equations are smooth up to dimension 9

  Abstract:The regularity of stable solutions to semilinear elliptic PDEs has been studied since the 1970's. It was initiated by a work of Crandall and Rabinowitz, motivated by the Gelfand problem in combustion theory. The theory experienced a revival in the mid-nineties after new progress made by Brezis and collaborators. I will present these developments, as well as a recent work, in collaboration with Figalli, Ros-Oton, and Serra, which finally establishes the regularity of stable solutions up to the optimal dimension 9. I will also describe a more recent paper of mine which provides full quantitative proofs of the regularity results.

- **Zhijie Chen** (Tsinghua University)

  Title: Asymptotic behaviors of low energy nodal solutions for Lane-Emden problems

  Abstract: In this talk, I will introduce our recent work about asymptotic behaviors of low energy nodal solutions for Lane-Emden equations when the exponent goes to infinity. Aymptotics of positive solutions have been studied well, but asymptotics of nodal solutions is more difficult to study and not much is known. Here we can obtain some results for low energy nodal solutions. This is based on joint work with my students Zetao Cheng and Hanqing Zhao.

- **Alessio Figalli** (ETH Zurich)

  Title: Quantitative stability in geometric and functional inequalities

  Abstract: Geometric and functional inequalities play a crucial role in several problems arising in analysis and geometry. Proving the validity of such inequalities, and understanding the structure of minimizers, is a classical and important question. In this talk, I will overview this beautiful topic and discuss some recent results.

- **Nicola Fusco** (Universita di Napoli "Federico II")

  Title:Local and global minimizers for a capillarity type problem

  Abstract:I will present a model for vapor-liquid-solid growth of nanowires where liquid drops are described as local or global volume-constrained minimizers of the capillarity energy outside a semi-infinite convex obstacle modeling the nanowire. I will first discuss global existence of minimizers and then, in the case of rotationally symmetric nanowires, I will explain how the presence of a sharp edge affects the shape of local minimizers and the validity of Young's law. Finally, I will present some recent regularity results for local minimizers and the connections of this problem with an isoperimetric inequality outside convex sets.

- **Jose A. Galvez** (Universidad de Granada)

  Title: Linearity of homogeneous solutions to elliptic equations in dimension three

  Abstract:An old conjecture by Alexandrov, Koutrofiotis and Nirenberg states that every 1-homogeneous solution to a linear elliptic equation in Euclidean 3-space must be linear. A striking counterexample to this claim was found by Martinez-Maure in 2001. In it, the Hessian of the solution vanishes exactly at 4 disjoint geodesic semicircles of the unit sphere, and along them the equation is not uniformly elliptic. In this talk we prove the converse of this result: for any (non-linear) homogeneous solution of a linear elliptic equation in Euclidean 3-space, there must exist four disjoint geodesic semicircles in the unit sphere along which the Hessian of u vanishes, and the uniform ellipticity of the equation is lost. The result is sharp, by Martinez-Maure's example. Joint work with Pablo Mira.

- **Yuxin Ge** (University of toulouse 3)

Title: Compactness of asymptotically hyperbolic Einstein manifolds in dimension 4 and applications

Abstract: Given a closed riemannian manfiold of dimension 3 $(M^3, [h])$, when will we fill in an asymptotically hyperbolic Einstein manifold of dimension 4 (X^4, g_+) such that $r^2 g_+|_M = h$ on the boundary $M = \partial X$ for some defining function r on X^4 ? This problem is motivated by the correspondance AdS/CFT in quantum gravity proposed by Maldacena in 1998 et comes also from the study of the structure of asymptotically hyperbolic Einstein manifolds. In this talk, I discuss the compactness issue of asymptotically hyperbolic Einstein manifolds in dimension 4, that is, how the compactness on conformal infinity leads to the compactness of the compactification of such manifolds under the suitable conditions on the topology and on some conformal invariants. As application, I discuss the uniqueness problem and non-existence result. It is based on the works with Alice Chang.

- **Genggeng Huang** (Fudan University)

Title: Monge-Ampere equation with Guillemin boundary condition

Abstract: We will talk about the following boundary value problem of Monge-Ampere equation

$$\det D^2 u = \frac{h(x)}{\prod_{i=1}^N l_i(x)}, \quad \text{in} \quad P \subset \mathbb{R}^n, \tag{1}$$

$$u(x) - \sum_{i=1}^N l_i(x) \log l_i(x) \in C^\infty(\overline{P}) \tag{2}$$

Here

$$0 < h(x) \in C^\infty(\overline{P}), \quad P = \cap_{i=1}^N \{l_i(x) > 0\}$$

is a simple convex polytope in $\mathbb{R}^n$. $l_i(x)$ are affine functions $i = 1, \cdots, N$. Under suitable conditions, we will show (1) and (2) are solvable. This is a joint work with Weiming Shen.

- **Qirui Li** (Zhejiang University)

Title: On the L_p Minkowski problem with super critical exponents.

Abstract: The L_p-Minkowski problem deals with the existence of closed convex hypersurface with prescribed p-area measure. The problem has been solved in the sub-critical case $p > -n - 1$, but remains widely open

in the super-critical case $p < -n-1$. In this talk, we introduce new ideas to solve the problem for all super-critical exponents. A crucial ingredient in the proof is a topological method based on the calculation of the homology of a topological space of ellipsoids. The talk is based on recent joint work with Qiang Guang and Xu-Jia Wang.

- **Emanuel Milman** (Technion-Israel Institute of Technology)

  Title: Multi-Bubble Isoperimetric Problems - Old and New

  Abstract:The classical isoperimetric inequality in Euclidean space $\mathbb{R}^n$ states that among all sets of prescribed volume, the Euclidean ball minimizes surface area. One may similarly consider isoperimetric problems for more general metric-measure spaces, such as on the n-sphere $\mathbb{S}^n$ and on n-dimensional Gaussian space $\mathbb{G}^n$ (i.e. $\mathbb{R}^n$ endowed with the standard Gaussian measure). Furthermore, one may consider the "multi-bubble" isoperimetric problem, in which one prescribes the volume of $p \geq 2$ bubbles (possibly disconnected) and minimizes their total surface area – as any mutual interface will only be counted once, the bubbles are now incentivized to clump together. The classical case, referred to as the single-bubble isoperimetric problem, corresponds to $p = 1$; the case $p = 2$ is called the double-bubble problem, and so on.

  In 2000, Hutchings, Morgan, Ritoré and Ros resolved the double-bubble conjecture in Euclidean space $\mathbb{R}^3$ (and this was subsequently resolved in $\mathbb{R}^n$ as well) – the boundary of a minimizing double-bubble is given by three spherical caps meeting at 120°-degree angles. A more general conjecture of J. Sullivan from the 1990's asserts that when $p \leq n + 1$, the optimal multi-bubble in $\mathbb{R}^n$ (as well as in $\mathbb{S}^n$) is obtained by taking the Voronoi cells of $p + 1$ equidistant points in $\mathbb{S}^n$ and applying appropriate stereographic projections to $\mathbb{R}^n$ (and backwards).

  In 2018, together with Joe Neeman, we resolved the analogous multi-bubble conjecture for $p \leq n$ bubbles in Gaussian space $\mathbb{G}^n$ – the unique partition which minimizes the total Gaussian surface area is given by the Voronoi cells of (appropriately translated) $p + 1$ equidistant points. In the present talk, we describe our recent progress with Neeman on the multi-bubble problem on $\mathbb{R}^n$ and $\mathbb{S}^n$. In particular, we show that minimizing bubbles in $\mathbb{R}^n$ and $\mathbb{S}^n$ are always spherical when $p \leq n$, and we resolve the latter conjectures when in addition $p \leq 5$ (e.g. the triple-bubble conjectures when $n \geq 3$ and the quadruple-bubble conjectures when $n \geq 4$).

- **Xavier Ros Oton** (Universitat de Barcelona)

Title: The singular set in the Stefan problem

Abstract: The Stefan problem, dating back to the XIXth century, is probably the most classical and important free boundary problem. The regularity of free boundaries in the Stefan problem was developed in the groundbreaking paper (Caffarelli, Acta Math. 1977). The main result therein establishes that the free boundary is C^∞ in space and time, outside a certain set of singular points. The fine understanding of singularities is of central importance in a number of areas related to nonlinear PDEs and Geometric Analysis. In particular, a major question in such a context is to establish estimates for the size of the singular set. The goal of this talk is to present some recent results in this direction for the Stefan problem. This is a joint work with A. Figalli and J. Serra.

- **Yannick Sire** (Johns Hopkins University)

Title: Geometric variational problems: regularity vs singularity formation

Abstract: I will describe in a very informal way some techniques to deal with the existence (and more qualitatively regularity vs singularity formation) in different geometric problems and their heat flows motivated by (variations of) the harmonic map problem, the construction of Yang-Mills connections or nematic liquid crystals. I will emphasize in particular on recent results on the construction of very fine asymptotics of blow-up solutions via a new gluing method designed for parabolic flows. I'll describe several open problems and many possible generalizations, since the techniques are rather flexible.

- **Guofang Wang** (University of Freiburg)

Title: Optimal geometric inequalities for capillary hypersurfaces

Abstract: In the talk I will first review our previous work on hypersurfaces with free boundary supported on the unit sphere. Then I will introduce suitable geometric quantities, quermassintegrals, for capillary hypersurfaces supported on a hyperplane and consider the corresponding Alexandrov-Fenchel inequalities by introducing a suitable curvature flow. I will also talk about a corresponding Heintze-Karcher-Ros inequality and a Minkowski problem. The talk bases on the joint work with Chao Xia and other collaborators.

- **Kelei Wang** (Wuhan University)

Title: Nondegeneracy for stable solutions to one phase free boundary problem

Abstract: Since the seminal work of Alt-Caffarelli in 1981, the one phase free boundary problem has been studied by many people. To study the regularity and singularity of free boundaries, the blow up analysis is a standard method. It turns out for this free boundary problem, the nondegeneracy condition is crucial for the application of this method. Although the nondegeneracy condition has been known for energy minimizers for a long time, it's not true for general solutions. In this talk, I will discuss a proof of the nondegeneracy for stable solutions. This is based on a joint work with N. Kamburov.

- **Xu-jia Wang** (Australian National University)

Title: Free boundary problems in the Monge-Ampere equation

Abstract: In this talk we consider the regularity of free boundary in the Monge- Ampere obstacle problem, and the regularity of free boundary in the Gauss curvature flow of convex hypersurface with flat side. By the Legendre transform, these problems are equivalent to the regularity of solutions to Monge-Ampere type equations with a singular point in polar coordinates. By analysing the geometric profile carefully near the singular point, we prove the $C^{2,\alpha}$ regularity for the free boundary in all dimensions.

- **Zhizhang Wang** (Fudan University)

Title: Hessian equations on exterior domains in hyperbolic space

Abstract: Suppose Ω is some domain in the hyperbolic space $\mathbb{H}^n$. In this talk, we will consider the homogenous k-Hessian equations on $\mathbb{H}^n \setminus \Omega$ with constant -1 on the boundary of Ω and asymptotic to zero at the infinity. We will give the existence of this equation. This is a joint work with Ling Xiao.

- **Yong Wei** (University of Science and Technology of China)

Title: Curvature measures and volume preserving curvature flows

Abstract: Volume preserving mean curvature flow was introduced by Huisken in 1987 and it was proved that the flow deforms convex initial hypersurface smoothly to a round sphere. This was generalized later by McCoy in 2005 and 2017 to volume preserving flows driven by a large class

of 1-homogeneous symmetric curvature functions. In this talk, we discuss the flows with higher homogeneity and describe the convergence result for volume preserving curvature flows in Euclidean space by arbitrary positive powers of k-th mean curvature for all $k = 1, \cdots, n$. As key ingredients, the monotonicity of a generalized isoperimetric ratio will be used to control the geometry of the evolving hypersurfaces and the curvature measure theory will be used to prove the Hausdorff convergence to a sphere. We also discuss some generalizations including the flows in the anisotropic setting, and the flows in the hyperbolic setting. The talk is based on joint work with Ben Andrews (ANU), Yitao Lei (ANU), Changwei Xiong (Sichuan Univ.), Bo Yang (CAS) and Tailong Zhou (USTC).

- **Jingang Xiong** (Beijing Normal University)

  Title: Harmonic maps with finite hyperbolic distances to the Extreme Kerr

  Abstract: Motivated by stationary vacuum solutions of the Einstein field equations, we study singular harmonic maps from domains of 3-dimensional Euclidean space to the hyperbolic plane having bounded hyperbolic distance to Kerr harmonic maps. In the degenerate case, we prove that every such harmonic map admits a unique tangent harmonic map at the extreme black hole horizon. The possible tangent maps are classified and rates of convergence to the tangent map are established. Similarly, expansions in the asymptotically flat end are presented. These results, together with those of Li-Tian 1992 and Weinstein 1989, provide a complete regularity theory for such singular harmonic maps. This is joint with Q. Han, M. Khuri and G. Weinstein.

- **Jie Zhou** (Capital Normal University)

  Title: Regularity for varifolds with critical Allard conditions

  Abstract: The classical Allard regularity theorem says, for a rectifiable n-varifold in the unit ball of the Euclidean space passing through the origin with density not less than one, if its the mass in the unit ball is close to the volume of a flat n-dimensional unit disk and the L^p norm of the generalized mean curvature is small enough for some supercritical index $p > n$, then the support of the varifold is a $C^{1,\alpha=1-\frac{n}{p}}$ graph near the origin. In this talk, we will present some regularity result in the critical case. In dimension two, we show the support of the varifold is (locally) bi-Lipschitz homeomorphic to the unit disk. In dimension $n > 2$, we discuss the $W^{1,p}$ regularity for $p < \infty$. The presentation is based on joint works

with Dr. Yuchen Bi.

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3 Participants

No.	Participant	Affiliation
1	Karoly Boroczky	Renyi Institute of Mathematics
2	Xavier Cabre	ICREA and Universitat Politecnica de Catalunya
3	Chuanqiang Chen	Ningbo University
4	Ruosi Chen	Tsinghua University
5	Shibing Chen	University of Science and Technology of China
6	Zhijie Chen	Tsinghua University
7	Bin Deng	Wuhan University
8	Hao Fang	The University of Iowa
9	Alessio Figalli	ETH Zurich
10	Nicola Fusco	Universita di Napoli "Federico II"
11	Jose Galvez	Universidad de Granada
12	Jing Gao	University of Science and Technology of China
13	Zhenghuan Gao	Shanghai University
14	Yuxin Ge	Universite Paul Sabatier
15	Bangxian Han	University of Science and Technology of China
16	Shengnan Hu	University of Science and Technology of China
17	Genggeng Huang	Fudan University
18	Inigo	Universitat Politecnica de Catalunya
19	Huaiyu Jian	Tsinghua University
20	Wangjian Jian	Academy of mathematics and systems science, CAS
21	Chunhe Li	University Of Electronic Science And Technology Of China
22	Jiahuan Li	University of Science and Technology of China
23	Li Li	Academy of mathematics and systems science, CAS
24	Qirui Li	Zhejiang University
25	Tongtong Li	Academy of mathematics and systems science, CAS
26	Xiang Li	Zhejiang Normal University
27	Yuanyuan Li	University of Science and Technology of China
28	Weizhao Liang	University of Science and Technology of China
29	Genqian Liu	Beijing Institute of Technology
30	Weiru Liu	University of Science and Technology of China
31	Biao Ma	Beijing International Center For Mathematical Research
32	Xinan Ma	University of Science and Technology of China
33	Emanuel Milman	Technion-Israel Institute of Technology
34	Quochung Nguyen	Academy of mathematics and systems science, CAS
35	Xavier Ros-Oton	Universitat de Barcelona
36	Qianzhong Ou	Guangxi Normal University
37	Guohuan Qiu	Academy of mathematics and systems science, CAS

38	Renzo	Universitat Politecnica de Catalunya
39	Shujun Shi	Harbin Normal University
40	Yannick Sire	Johns Hopkins University
41	Liming Sun	Academy of mathematics and systems science, CAS
42	Xushan Tu	The Hong Kong University of Science and Technology
43	Guofang Wang	University of Freiburg
44	Kelei Wang	Wuhan University
45	Xu-jia Wang	Australian National University
46	Youde Wang	Academy of mathematics and systems science, CAS
47	Zhiwei Wang	University of Science and Technology of China
48	Zhizhang Wang	Fudan University
49	Wei Wei	Nanjing University
50	Yong Wei	University of Science and Technology of China
51	Ke Wu	Wuhan University
52	Tian Wu	University of Science and Technology of China
53	Wangzhe Wu	University of Science and Technology of China
54	Yantao Wu	Johns Hopkins University
55	Shaofen Xie	Academy of mathematics and systems science, CAS
56	Jingang Xiong	Beijing Normal University
57	Lei Xu	University of Science and Technology of China
58	Lu Xu	Hunan University
59	Shuning Xu	University of Science and Technology of China
60	Jin Yan	University of Science and Technology of China
61	Jianmin Yang	University of Science and Technology of China
62	Wen Yang	Wuhan Institute of Physics and Mathematics,CAS
63	Jian Ye	Academy of mathematics and systems science, CAS
64	Bao Yu	University of Science and Technology of China
65	Dekai Zhang	Shanghai University
66	Liqun Zhang	Academy of mathematics and systems science, CAS
67	Ping Zhang	Academy of mathematics and systems science, CAS
68	Xinyun Zhang	Zhejiang Normal University
69	Yi Zhang	Academy of mathematics and systems science, CAS
70	Jie Zhou	Capital Normal University
71	Jundong Zhou	Fuyang Normal University
72	Xiao Zhou	University of Science and Technology of China
73	Xin Zhou	Cornell University
74	Yang Zhou	University of Science and Technology of China
75	Yifu Zhou	Wuhan University
76	Yuan Zhou	Beijing Normal University
77	Hua Zhu	University of Science and Technology of China
78	Jingyong Zhu	Sichuan University

4 Acknowledgments

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5 Useful information

5.1 Transportation and Accomodation

Airport Shuttle

1. Beijing Capital International Airport:

Take airport shuttle line 5 to Zhongguancun Station which costs about 30 Yuan.

2. Beijing Daxing International Airport:

Take Daxing airport line to Caoqiao Station and transfer to subway line 10 to Zhichunlu or Zhichunli Station which costs about 40 Yuan.

Subway

Take subway line 10 to Zhichunlu or Zhichunli Station which costs about 30 Yuan from airport or 5 Yuan from train station.

Web-page: http://www.bjsubway.com/en/

Taxi

Take taxi to Baofusiqiao which costs about 100 Yuan from airport or 50 Yuan from train station.

Accommodation
1. Liaoning International Hotel 5-star

Address: No.2 A North 4th Ring Road West, Haidian District, Beijing

Telephone: 8610-62589999

Web-page: http://www.liaoninginternationalhotel.com/

2. Wuke Hotel 3-star

Address: No.55, Zhongguancun East Rd., Haidian District, Beijing

Telephone: 8610-82649140

Web-page: http://www.bjwkhotel.com/

3. Scientist Hotel 2-star

Address: No.18, West Road, North Fouth Ring, Haidian District

Telephone: 8610-82628538

4. Beijing Jingyi Hotel 4-star

Address: No.9 Dazhongsi East Road, Haidian District, Beijing

Telephone: 8610-62165588

5.2 Contact us

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- Yi Zhang, email: yzhang@amss.ac.cn

6 Brief introduction of AMSS

Academy of Mathematics and Systems Science (AMSS) was founded in December 1998, by merging the Institute of Mathematics, Institute of Applied Mathematics,

Institute of Systems Science, and Institute of Computational Mathematics and Scientific/Engineering Computing of CAS. Its history can be traced back to the founding of the Institute of Mathematics of CAS in 1952.

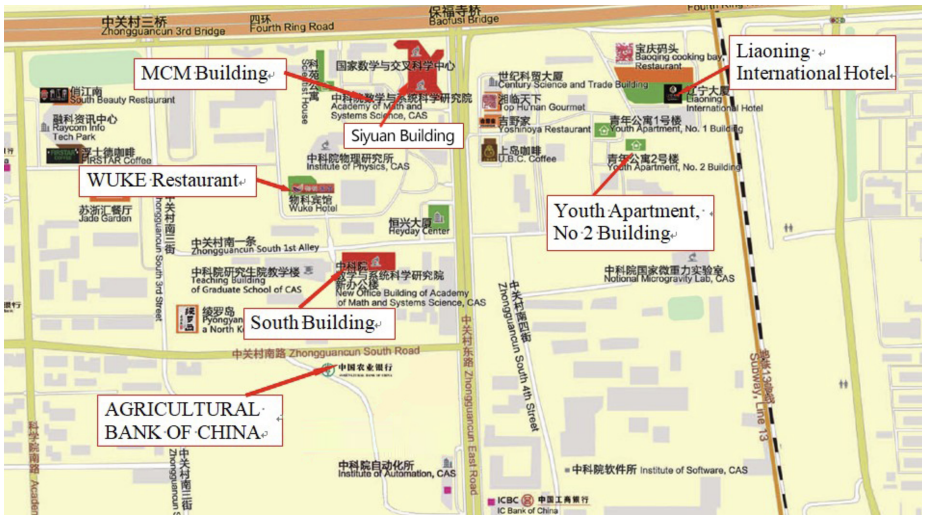
CAS is the top academic research organization of P. R. China. This defines the mission of the AMSS: pursuing fundamental research of highest level on mathematical sciences and their applications and related interdisciplinary study; high level training in mathematical sciences; providing consulting reports to the authorities on certain issues related to mathematical sciences and their applications; playing a leading role in advanced study of mathematics in China; and becoming a mathematical research center of global influence.

The AMSS is the largest, strongest and the most influential research institution in mathematical science in China. Its research areas are comprehensive, ranging from mathematics, statistics, systems science, management science, computer science. Now AMSS is dynamic in following subjects: number theory, algebra, geometry and topology, analysis and mathematical physics, scientific computing, probability and statistics, systems and control, operations research and management science, and computer mathematics. Currently AMSS has 222 faculty members, including 118 professors (17 are academicians), 81 associate professors, and 27 assistant professors.

To train young people is also a major obligation of the AMSS. At the moment, it has more than 70 postdoctoral fellows, 425 PhD students and 305 postgraduate students. Meanwhile, AMSS is engaged in the undergraduate education at the University of Chinese Academy of Sciences by offering all mathematical courses and tutoring.

The AMSS has a brilliant history and an outstanding status, and is full of vigour and vitality now. Outstanding young people are welcome to join the AMSS. Prospective students are welcome to continue their study at the AMSS as graduate students. People here would keep pure mind and follow higher standard to pursue new heights of excellence along with the AMSS.

Map of the campus



Offline session will be held at **South Building 202 room**, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing.